Evaluating Path Costs in Multi-Attributed Fuzzy Weighted Graphs

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Abstract— In this paper, we consider the problem of choosing a least-cost path from a graph that is attributed with multiple fuzzy weights. The cost of a path is determined by multiple conflicting objectives that seek to minimize either the total or maximum values of each feature over the length of the path. We present a framework for evaluating paths with various agent preferences. Our method allows the agent to pick any Pareto optimal path and can be used within a larger framework to model decision-making behavior. Our approach is demonstrated on a hand-crafted example problem.

Keywords—fuzzy decision-making, multiobjective optimization, shortest path problem, non-additive objective

I. Introduction

Pathfinding is a critical component of mobile autonomous agents. Given an environment space $\mathcal{E}$, the agent must decide how best to navigate from its current location to some destination or may even need to determine where to go in the first place. In practice, the representation of the environment plays an important factor in how well the agent can operate. Typically, the environment is treated as a graph $G(\mathcal{E})$, where vertices are valid locations and edges indicate the cost of travel. Agents with multiple objectives to satisfy may assign a vector of weights to each edge so that any path can be evaluated in terms of each objective. In crisp domains, this graph is known completely and can be used by the agent to construct a plan. However, when the environment is not known exactly, some uncertainty must be captured by the graph representation. Multi-attributed fuzzy weighted graphs can be used to describe these problems and find solutions that best satisfy the agent's objectives.

Consider the example problem given in Fig. 1, where an agent needs to choose a path through the environment graph from vertex 1 to 5. Each edge of the graph is annotated with a distance and slope attribute, described using linguistic terms and defined as triangular fuzzy numbers. The agent has two objectives: minimize both the total path distance and the maximum slope along any single edge. This is an instance of the multiobjective fuzzy least-cost path problem (MO-FLCPP), which can have different solutions depending on the agent's preferences. We will present in this paper a method for modeling the agent's preferences and choosing a solution path from the Pareto optimal set.

This work is motivated by the desire to model various agent behaviors in fixed environments. For instance, in the example problem, one agent may prefer to take the shortest route even though it goes over a steep hill, while another agent might choose to take the longer but flatter path. By understanding how the agent evaluates and ranks each of the available options, we may begin to build predictive models of the agent's behavior. This lets us answer questions such as, "how would this agent behave in a different environment?" To study these types of problems, we developed the computational mental map (CMM) framework [1], which implements the method presented here for evaluating MO-FLCPPs as part of the broader problem of developing agents that can demonstrate various behaviors and strategies in uncertain and partially observable environments. The CMM framework can generate a wide variety of benchmark problems such as the one shown in Fig. 2, and although there are various approaches to choosing a goal location and finding the best path to get there, our focus in this paper is on how the agent can evaluate various path options that have already been provided and choose among them.

We begin in Section II by defining the representations for fuzzy numbers and fuzzy weighted graphs. Then Section III shows how multiobjective optimization methods can be applied to multi-attributed fuzzy weighted graphs. Section IV gives our conclusions and directions for future work.

II. Multi-Attributed Fuzzy Weighted Graphs

A. Fuzzy Numbers

A fuzzy number $A \subseteq \mathbb{R}$ is a normalized convex fuzzy set with a membership function $\mu_A: A \rightarrow [0, 1]$ that specifies the degree to which a real number $x \in \mathbb{R}$ is included in the set A. Fuzzy numbers provide a way to represent uncertainty in the true value of a number and to express linguistic approximations such as "about 3" or "nearly 10." We use triangular fuzzy numbers throughout this work to demonstrate our approach, but other representations (such as trapezoids or alpha-cuts) could be used with minor modifications. A triangular fuzzy number A is defined by a 3-tuple $\text{Tri}(a, b, c)$, where the interval $[a, c]$

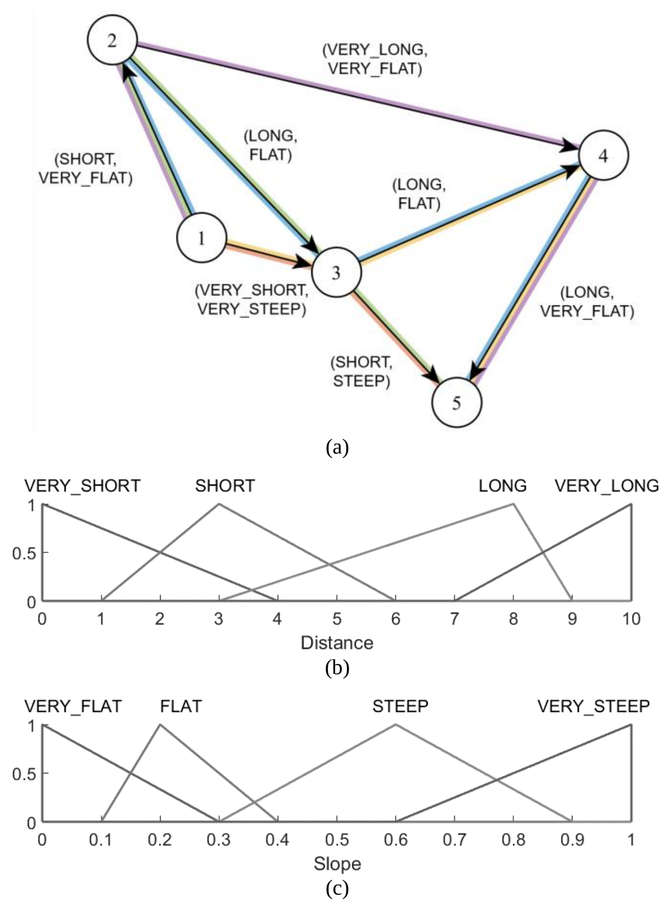

Fig. 1. An example fuzzy weighted graph (a) with two features per edge: distance (b) and slope (c), represented as triangular fuzzy numbers. There are five unique paths between the vertices 1 and 5, colored red, yellow, green, blue, and purple (described in Table I).

is the support for which $\mu_A(x) > 0$ and b is the single point where $\mu_A(x) = 1$.

The arithmetic operators $(+, -, \times, \div)$, as well as other functions such as minimization and maximization, can be defined for fuzzy numbers using Zadeh's extension principle. The result of a function $f(A, B)$ operating on two fuzzy numbers A and B is given as

$$\mu_{f(A,B)}(z) = \sup_{z=f(x,y)} \min\big(\mu_A(x), \mu_B(y)\big). \quad (1)$$

In this paper, we focus on the summation and maximization operators for triangular fuzzy numbers. The summation of two triangular fuzzy numbers is derived from (1) as

$$\begin{aligned} &\mathrm{Tri}(a_1, b_1, c_1) + \mathrm{Tri}(a_2, b_2, c_2) \\ &\quad = \mathrm{Tri}(a_1 + a_2, b_1 + b_2, c_1 + c_2). \end{aligned} \quad (2)$$

The summation of any two triangular fuzzy numbers will always result in a new triangular fuzzy number. However, because maximization is a nonlinear operator, the maximum of two triangular fuzzy numbers may not be triangular (see Fig. 3).

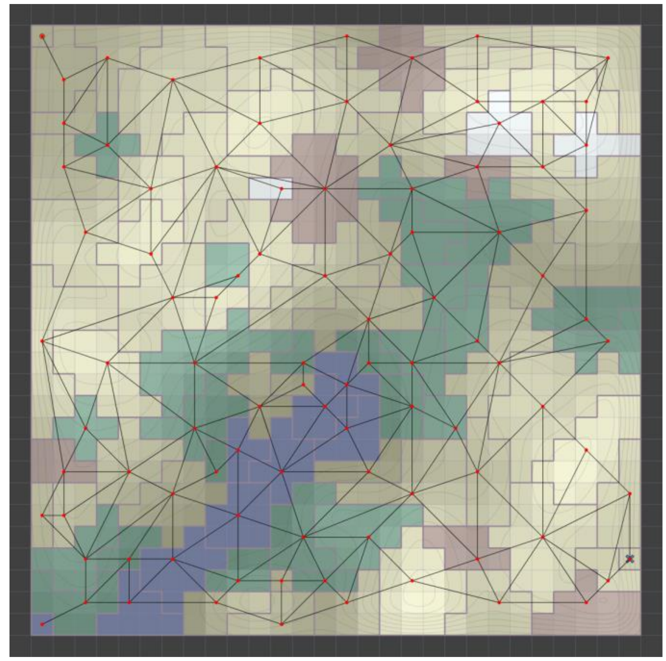

Fig. 2. Screenshot of a fuzzy weighted graph created within the CMM framework from a randomly generated simulated environment. The graph connects adjacent regions and is attributed with fuzzy features such as distance, type of terrain, and elevation.

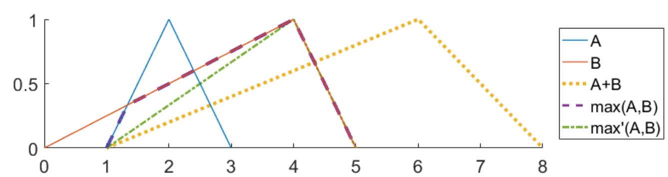

Fig. 3. The summation of two triangular fuzzy numbers $A = \mathrm{Tri}(1, 2, 3)$ and $B = \mathrm{Tri}(0, 4, 5)$ is shown as $\mathrm{Tri}(1, 6, 8)$. The true maximum of A and B is not a triangular fuzzy number but can be approximated as $\mathrm{Tri}(1, 4, 5)$.

To keep the practical computational requirements of our method simple, we seek to maintain a consistent representation for all fuzzy numbers. Therefore, we approximate the result of the maximization operator as a new triangular fuzzy number,

$$\begin{aligned} &\max'\big(\mathrm{Tri}(a_1, b_1, c_1), \mathrm{Tri}(a_2, b_2, c_2)\big) \\ &= \mathrm{Tri}(\max(a_1, a_2), \max(b_1, b_2), \max(c_1, c_2)). \end{aligned} \quad (3)$$

This approach maintains the true definition at the endpoints and peak of the fuzzy number but may produce different values at intermediate points.

In a least-cost path problem, the goal is to find a solution path that minimizes some set of objectives. By representing the value of a solution as a fuzzy number, we can capture some of the uncertainty in a solution's true value. However, this uncertainty makes it difficult to assess whether one solution is better than another (smaller aggregated path cost being better). While there is no universal definition for the ordering of fuzzy numbers that proves satisfactory in all cases (see for instance [2], [3]), we adopt the following intuitive definitions. Let $A_1 = \mathrm{Tri}(a_1, b_1, c_1)$ and $A_2 = \mathrm{Tri}(a_2, b_2, c_2)$ be two triangular fuzzy numbers. We say that A_1 is less than or equal to A_2 ($A_1 \leq A_2$) if and only if ($a_1 \leq a_2$ and $b_1 \leq b_2$ and $c_1 \leq c_2$). We say that A_1 is strictly less than A_2 ($A_1 < A_2$) if and only if $A_1 \leq A_2$ and ($a_1 < a_2$ or $b_1 < b_2$ or $c_1 < c_2$). If $A_1 \nleq A_2$ and $A_2 \nleq A_1$, then it is not clear which of the two fuzzy numbers should be preferred (assuming $A_1 \neq A_2$).

There may be many solutions for a given problem with no single solution that is less than all the others. When a decision-maker is required to choose one of these, we employ a weighted centroid defuzzification scheme to produce a crisp value for each solution that can be ranked directly. The centroid of a fuzzy number A is defined as

$$\bar{x} = \frac{\int x\mu_A(x)\,dx}{\int \mu_A(x)\,dx}. \tag{4}$$

For a triangular fuzzy number $\mathrm{Tri}(a, b, c)$, this evaluates to

$$\bar{x} = \frac{1}{3}(a + b + c). \tag{5}$$

The weighted centroid is defined by a control parameter $\xi \in [0, 1]$ that specifies the optimism/pessimism of the decision-maker. A value of $\xi = 0$ indicates extreme optimism, in which the fuzzy number is defuzzified to the smallest possible value, a. A value of $\xi = 1$ indicates extreme pessimism, where defuzzification results in the largest possible value, c. A value of $\xi = 0.5$ gives a balanced approach using the centroid, $\bar{x}$. The crisp weighted centroid value is linearly interpolated between these points as

$$C(A|\xi) = \begin{cases} a + 2\xi(\bar{x} - a), & \xi \leq 0.5 \\ \bar{x} + 2(\xi - 0.5)(c - \bar{x}), & \xi > 0.5\,. \end{cases} \tag{6}$$

Using a constant value for ξ, a decision-maker can defuzzify multiple fuzzy numbers using the weighted centroid approach and compare the resulting crisp values. Smaller values indicate better solutions (optimistic), whereas larger values are worse (pessimistic). If two fuzzy numbers result in the same crisp value when defuzzified, they are considered equivalent.

B. Fuzzy Weighted Graphs

A physical environment or problem space can be represented as a graph G with vertex set $V(G)$ and edge set $E(G)$. Each vertex $v \in V(G)$ represents a location or state, and edges represent possible actions or movements between locations. In a directed graph, the edge set $E(G) \subseteq V(G) \times V(G)$ consists of all ordered pairs of vertices (v_s, v_t) that are connected by an edge. An edge $e \in E(G)$ has both a starting vertex $v_s = \mathrm{START}(e)$ and an ending vertex $v_t = \mathrm{END}(e)$. A path p through the graph is an n-tuple $(e_1, \dots, e_n) \in \left(E(G)\right)^n$ where $\mathrm{END}(e_i) = \mathrm{START}(e_{i+1})$ for $i = 1, \dots, n-1$. The starting and ending vertices of the path are denoted as $s = \mathrm{START}(e_1)$ and $t = \mathrm{END}(e_n)$ respectively. $P(s,t)$ is the set of all paths between vertices s and t.

TABLE I
AGGREGATED FEATURE VALUES OF THE EXAMPLE GRAPH

Path	Color	Total Distance	Max Slope
1-3-5	Red	$\mathrm{Tri}(1, 3, 10)$	$\mathrm{Tri}(0.6, 1, 1)$
1-3-4-5	Yellow	$\mathrm{Tri}(6, 16, 22)$	$\mathrm{Tri}(0.6, 1, 1)$
1-2-3-5	Green	$\mathrm{Tri}(5, 14, 21)$	$\mathrm{Tri}(0.3, 0.6, 0.9)$
1-2-3-4-5	Blue	$\mathrm{Tri}(10, 27, 33)$	$\mathrm{Tri}(0.1, 0.2, 0.4)$
1-2-4-5	Purple	$\mathrm{Tri}(11, 21, 25)$	$\mathrm{Tri}(0, 0, 0.3)$

In a fuzzy weighted graph, each edge e is assigned a weight vector $\boldsymbol{F}(e) = \left(F_1(e), \dots, F_m(e)\right)$, where each $F_i(e)$ is a fuzzy number representing a feature attribute of that edge. Features are defined as components of a multiobjective cost function that are intended to be minimized. For instance, a weight vector might have one feature that represents distance and another that represents slope or travel time. Each feature is assumed to be non-negative with zero being the minimum possible value. By using a vector of fuzzy numbers to represent edge weights, a fuzzy weighted graph can model different degrees of uncertainty for each component of the multiobjective cost function.

A path $p = (e_1, \dots, e_n)$ in a fuzzy weighted graph is a sequence of edges, each with an associated weight vector. We compute an aggregated cost vector $\boldsymbol{A}(p) = \left(A_1(p), \dots, A_m(p)\right)$ for the path by either summing or taking the maximum value of the feature components of each edge. Let $\boldsymbol{\gamma} = (\gamma_1, \dots, \gamma_m)$ be an indicator vector where $\gamma_i = 0$ if feature i should be aggregated by summation and $\gamma_i = 1$ if feature i should be aggregated using maximization. For features where the decision-maker considers the total feature value ($\gamma_i = 0$), the aggregated value of feature i is

$$A_i(p) = \sum_{j=1}^{n} F_i(e_j). \tag{7}$$

For features where the decision-maker considers the maximum feature value ($\gamma_i = 1$), the aggregated value of feature i is

$$A_i(p) = \max_{j=1,\dots,n}{}' F_i(e_j). \tag{8}$$

Note that the aggregation method may be different for each feature. In the example in Fig. 1, the multiobjective cost function consists of a distance feature and a slope feature, where the decision-maker seeks to find a path with the shortest total distance and the smallest maximum slope. In this case, the distance feature is aggregated using summation, whereas the slope feature uses maximization. There are five unique paths between vertices 1 and 5 in the example graph. The aggregated feature values of the paths are given in Table I and are plotted in Fig. 4.

Given a fuzzy weighted graph G with a starting vertex $s \in V(G)$ and an ending vertex $t \in V(G)$ where $s \neq t$, the multiobjective fuzzy least-cost path problem (MO-FLCPP) is defined as finding a path $p \in P(s,t)$ that minimizes the

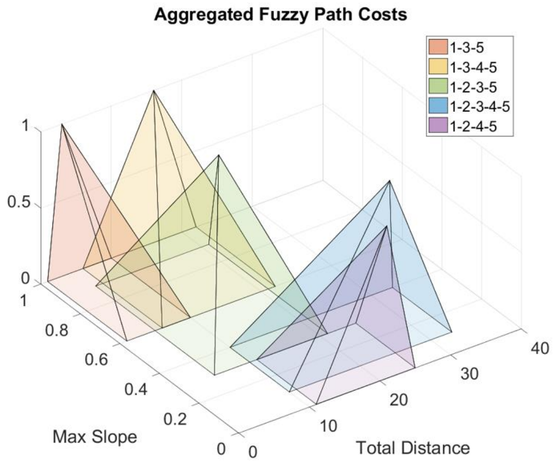

Fig. 4. Plots of the two-dimensional aggregated fuzzy cost vectors for each path in the example graph from Fig. 1.

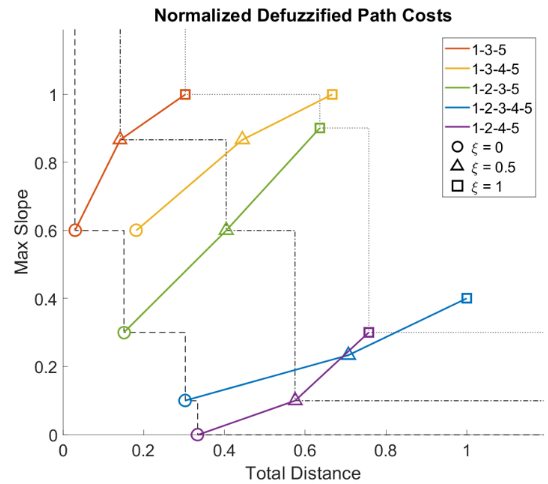

Fig. 5. The aggregated fuzzy cost vectors from Fig. 4 are normalized using the nadir vector and defuzzified using weighted centroid defuzzification. The black dotted lines show the Pareto fronts for different values of ξ.

aggregated cost vector $\boldsymbol{A}(p)$. When the summation operator is used for aggregation, this is called the shortest path problem [4]. When the max operator is used, it may be called the minimax path problem [5]. We use the term least-cost path to refer to the general case that may have mixed aggregation methods. The MO-FLCPP may not have a single solution that minimizes each cost component $A_i(p)$ simultaneously for $i = 1, \dots, m$. Multiobjective optimization techniques should therefore be used to help the decision-maker choose a solution.

III. Multiobjective Optimization

A. Pareto Optimal Paths

A multiobjective optimization problem (MOP) deals in the simultaneous minimization of more than one objective [6]. Following the previous notation, a MO-FLCPP is defined as

$$\begin{aligned} &\text{minimize} && \boldsymbol{A}(p) = \big(A_1(p), \dots, A_m(p)\big) \\ &\text{subject to} && p \in P(s,t), \end{aligned}$$

where $m \geq 2$. Each component $A_i(p)$ for $i = 1, \dots, m$ represents an objective that is to be minimized. The objectives are typically in conflict, such that the minimum value of one objective cannot be obtained without some tradeoff in the other objectives. Nevertheless, some solutions (paths) are clearly better than others. We say that a path p dominates path p' ($p \prec p'$) if and only if $A_i(p) \leq A_i(p')$ for all $i = 1, \dots, m$ and there exists a $j \in \{1, \dots, m\}$ such that $A_j(p) < A_j(p')$. A path that dominates another path is at least as good as the other path in all objectives and is better in at least one objective. A path that is not dominated by any other known solution is said to be Pareto optimal. Formally, the Pareto optimal set PS is defined as

$$PS = \{p \in P(s,t) \mid \{p' \in P(s,t) \mid p' \prec p\} = \emptyset\}. \tag{9}$$

The multiobjective cost vectors of the paths in PS define the Pareto front,

$$PF = \{\boldsymbol{A}(p) \mid p \in PS\}. \tag{10}$$

In the example graph, all paths except the yellow path (1-3-4-5) are members of the Pareto optimal set. The yellow path is dominated by both the red (1-3-5) and green (1-2-3-5) paths.

The native units of each objective may be incomparable, making it difficult to assess the relative value of each solution. To make the comparison between solutions meaningful, the original cost vectors are normalized into a unit hypercube. This ensures that each objective is treated equally. For instance, if the distance cost is measured in meters and the slope cost is measured as a percentage of some reference angle, the magnitudes of these two dimensions should be normalized before being compared. To normalize the vectors, the minimum value of each objective is defined as zero and the maximum value is defined by the reference point $\boldsymbol{z}^* = (z_1^*, \dots, z_m^*)$. Determining the optimal value of $\boldsymbol{z}^*$ is not a trivial task and the value that is chosen can greatly affect the resulting decision. Ideally, $\boldsymbol{z}^*$ should be the nadir vector of the Pareto front, in which each z_i^* is defined as

$$z_i^* = \max_{p \in PS'} c_{ip}, \tag{11}$$

where $A_i(p) = \text{Tri}\big(a_{ip}, b_{ip}, c_{ip}\big)$. Here, PS' is the current best approximation of the Pareto optimal set since the true set may be unknown. The normalized cost vectors are then computed as $\boldsymbol{A}'(p) = \big(A_1'(p), \dots, A_m'(p)\big)$, where

$$A_i'(p) = \text{Tri}\left(\frac{a_{ip}}{z_i^*}, \frac{b_{ip}}{z_i^*}, \frac{c_{ip}}{z_i^*}\right) \tag{12}$$

for each $i = 1, \dots, m$.

Since the example problem is small, the Pareto optimal set can be determined directly, and the reference point is evaluated as the nadir vector $\boldsymbol{z}^* = (33, 1)$, as these are the largest possible values of the aggregated distance and slope features. Fig. 5 shows each of the normalized cost vectors after applying weighted centroid defuzzification to each feature. (We typically wait until after scalarizing the cost vectors to apply defuzzification, but this example helps show the process.) The black dotted lines show the location of the Pareto front for different values of ξ. From this we can see that the yellow path is always dominated, whereas the blue path (1-2-3-4-5) is dominated by the purple path (1-2-4-5) when $\xi = 0.5$ and $\xi = 1$. The blue path is only Pareto optimal when the decision-maker is very optimistic (i.e. expects the true cost of each path segment to be small).

B. *Scalarization*

All solutions that are members of the Pareto optimal set would be rational choices for the decision-maker. Ultimately, however, a single solution must often be chosen. Typically, this is done using a scalarization function that reduces the multiobjective optimization problem into an optimization problem with a single objective. Given a multidimensional fuzzy cost vector $\boldsymbol{X} = (X_1, \dots, X_m)$ where each X_i is a fuzzy number, and an objective weight vector $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_m)$ where $\lambda_i \geq 0$ and $\sum_i \lambda_i = 1$ for $i = 1, \dots, m$, the scalarization function $g(\boldsymbol{X}|\boldsymbol{\lambda})$ reduces the cost vector $\boldsymbol{X}$ to a single fuzzy number. This value can then be used to rank and compare various alternatives, with smaller values being preferred. The scalarized MO-FLCPP is defined as

$$\begin{array}{ll} \text{minimize} & g(\boldsymbol{A}'(p)|\boldsymbol{\lambda}) \\ \text{subject to} & p \in P(s,t). \end{array}$$

The path p that minimizes the scalarized value of the normalized aggregated cost vector $\boldsymbol{A}'(p)$ is chosen as the preferred solution. The objective weight vector $\boldsymbol{\lambda}$ represents the relative importance of each objective to the decision-maker, with more important objectives receiving higher weights. We consider three different scalarization functions: weighted sum, Tchebycheff, and ordered weighted average.

One of the most common scalarization methods is the weighted sum, which maintains a linear relationship between the decision-maker's preferences and the scalarized cost value. This is defined as

$$g^{\text{ws}}(\boldsymbol{X}|\boldsymbol{\lambda}) = \sum_{i=1}^{m} \lambda_i X_i, \tag{13}$$

where the multiplication of a scalar λ and a triangular fuzzy number $\text{Tri}(a, b, c)$ is defined as $\text{Tri}(\lambda a, \lambda b, \lambda c)$. If the shape of the Pareto front is convex, then the weighted sum can be a good choice because every Pareto optimal solution can be made to have the lowest scalarized cost by changing the objective weight vector. However, if the shape of the Pareto front is non-convex, then there will always be some Pareto optimal solution that can never be chosen. For a detailed discussion on why this is so, please see [6].

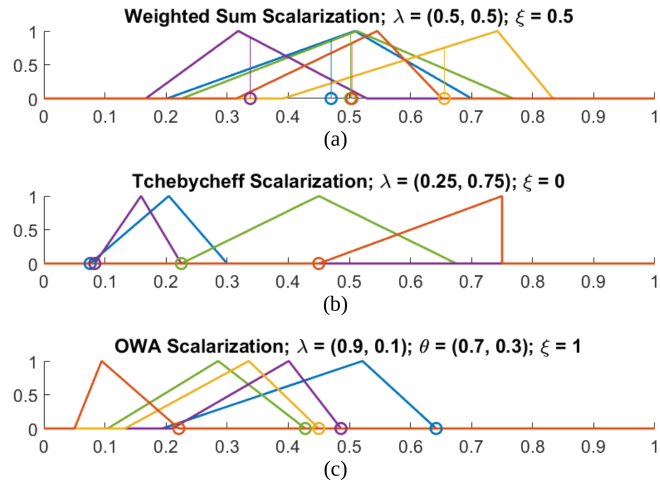

Fig. 6. Examples of different scalarization methods applied to the aggregated fuzzy cost vectors given in Table I. Each method represents a decision-maker with different preferences. The scalarized fuzzy numbers shown in the plots are defuzzified (shown as a circle and vertical line) and the decision-maker chooses the path with the lowest defuzzified cost.

A simple alternative to the weighted sum approach is the Tchebycheff method, which can be parameterized with different objective weight vectors to make any Pareto optimal solution have the lowest scalarized cost. The Tchebycheff scalarization function is defined as

$$g^{\text{te}}(\boldsymbol{X}|\boldsymbol{\lambda}) = \max_{i=1,\dots,m}{}' \; \lambda_i X_i. \tag{14}$$

This method evaluates the quality of a solution as the least satisfied weighted objective value. A single high cost for one objective can penalize an otherwise good solution.

The last scalarization approach we consider is based on the ordered weighted average operator (OWA) proposed by Yager [7]. This method requires the definition of an additional scalar weight vector $\boldsymbol{\theta} = (\theta_1, \dots, \theta_m)$ where $\theta_i \geq 0$ and $\sum_i \theta_i = 1$ for $i = 1, \dots, m$. Each θ_i represents the weighted contribution of the i^{th} largest scaled vector component. First, the cost vector $\boldsymbol{X}$ is scaled by the objective weight vector λ to give the scaled cost vector $\boldsymbol{Y} = (Y_1, \dots, Y_m)$, where $Y_i = \lambda_i X_i = \text{Tri}(a_i^Y, b_i^Y, c_i^Y)$ for $i = 1, \dots, m$. Next, we independently sort all the a_i^Y, b_i^Y, and c_i^Y values and define the lists $\left(a_{(1)}^Y, \dots, a_{(m)}^Y\right)$, $\left(b_{(1)}^Y, \dots, b_{(m)}^Y\right)$, and $\left(c_{(1)}^Y, \dots, c_{(m)}^Y\right)$, where $a_{(i)}^Y$, $b_{(i)}^Y$, and $c_{(i)}^Y$, are the i^{th} largest values in their respective lists. Once this is done, the OWA scalarization function is defined as

$$g^{\text{OWA}}(\boldsymbol{X}|\boldsymbol{\lambda}, \boldsymbol{\theta}) = \sum_{i=1}^{m} \theta_i \text{Tri}\left(a_{(i)}^Y, b_{(i)}^Y, c_{(i)}^Y\right). \tag{15}$$

The OWA scalarization method can be made to represent many different functions by changing the weight vector $\boldsymbol{\theta}$. For instance, the OWA operator behaves as the weighted sum when $\theta_i = \frac{1}{m}$ for all $i = 1, \dots, m$. (Although the ordering of solutions in this case is the same as the weighted sum, the actual values

TABLE II
BEST PATHS FOUND IN THE EXAMPLE GRAPH

		$\theta_1 =$	1	0.75	0.5	0.25	0
ξ	$\boldsymbol{\lambda}$	$\theta_2 =$	0	0.25	0.5	0.75	1
0	(0, 1)		P	P	P	P	–
	(0.25, 0.75)		B	P	P	P	P
	(0.5, 0.5)		G	P	P	P	P
	(0.75, 0.25)		G	G	R	R	P
	(1, 0)		R	R	R	R	–
0.5	(0, 1)		P	P	P	P	–
	(0.25, 0.75)		P	P	P	P	R
	(0.5, 0.5)		P	P	P	P	P
	(0.75, 0.25)		R	R	R	P	P
	(1, 0)		R	R	R	R	–
1	(0, 1)		P	P	P	P	–
	(0.25, 0.75)		P	P	P	P	R
	(0.5, 0.5)		P	P	P	P	P
	(0.75, 0.25)		R	R	R	P	P
	(1, 0)		R	R	R	R	–

may be different due to the additional scaling.) The Tchebycheff method is equivalent to setting $\theta_1 = 1$ and $\theta_i = 0$ for all $i \neq 1$. We can implement a form of bounded rationality [8], [9] for the decision-maker by defining a weight vector of the form $\theta_i = \frac{1}{q}$ for $i = 1, \ldots, q$ and $\theta_i = 0$ for all $i > q$. This represents the case where the decision-maker does not have the necessary computational resources to consider all objectives simultaneously and bases the decision on only the q least satisfied objective values.

To demonstrate the scalarization process, consider three different decision-makers that must choose a solution to the example problem. The first uses the weighted sum scalarization method with $\boldsymbol{\lambda} = (0.5, 0.5)$ and $\xi = 0.5$. Applying g^{ws} to each of the aggregated fuzzy path cost vectors in Table I gives the fuzzy values shown in Fig. 6a. The weighted centroid of each path is shown with a circle and a vertical line. The decision-maker chooses the path with the smallest defuzzified cost, which is the purple path. A different decision-maker using the Tchebycheff method with $\boldsymbol{\lambda} = (0.25, 0.75)$ and $\xi = 0$ computes the values shown in Fig. 6b. This is one of the few conditions where the blue path is evaluated as the lowest cost option. The last decision-maker shown in Fig. 6c uses the OWA scalarization method with $\boldsymbol{\lambda} = (0.9, 0.1)$, $\boldsymbol{\theta} = (0.7, 0.3)$ and $\xi = 1$. This represents extreme pessimism with a strong bias towards minimizing the distance feature, which results in giving the red path the lowest cost.

Table II shows the best paths found in the example graph for varying values of $\boldsymbol{\xi}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\theta}$ using OWA scalarization. Paths are notated with the first letter of their color. The first column where $\boldsymbol{\theta} = (1, 0)$ is equivalent to Tchebycheff scalarization and the middle column where $\boldsymbol{\theta} = (0.5, 0.5)$ is the same as the weighted sum method. The last column shows an edge case where $\boldsymbol{\theta} = (0, 1)$ indicating that the decision-maker considers only the most satisfied objective. When one of the objective weights is also 0, this feature becomes the same over all paths and so there is no single best path (indicated by a "–").

IV. CONCLUSIONS AND FUTURE WORK

Multiobjective problems such as the MO-FLCPP may not have a single optimal solution. Adding uncertainty in the form of fuzzy numbers further increases the number of ways a decision-making agent can choose a solution. In this paper, we presented a framework for modeling uncertain environment spaces as multi-attributed fuzzy weighted graphs and evaluating different path options. The agent is defined using several parameters which lead to unique solutions when varied. This shows the versatility of the model for capturing a wide range of agent behaviors.

The aggregation framework can handle both summation and maximization objectives. The nonlinearity of the maximization operator is unsuitable for many algorithms that would search a graph for a shortest path. However, a method is presented in [1] that decomposes the MO-FLCPP into a single-objective, crisp shortest path problem that can be solved using Dijkstra's algorithm. The resulting solution may be suboptimal but can be improved using a multiobjective evolutionary algorithm.

The CMM framework can be used to generate many different types of problems for studying agent decision-making behavior. In addition to the MO-FLCPP, problems with multiple goal locations such as the traveling salesman problem can be created and used with partially observable environments as in [10]. The decision to only use triangular fuzzy numbers arose as a performance consideration for the simulation environment, but the methods presented here could be extended to handle general LR fuzzy numbers.

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