An α-level OWA Implementation of Bounded Rationality for Fuzzy Route Selection

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Human Geography

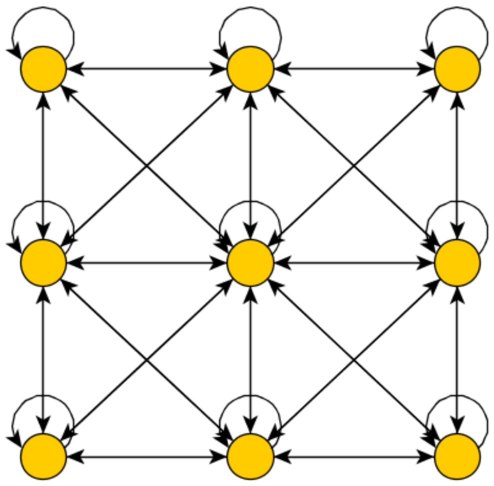
- Study of how people move and interact in geographic space
  - Cultural and individual differences give rise to different behaviors
- Agent-based modeling
  - Interaction with physical geography
  - Stochastic realization of complex decision making

Human Decision Making

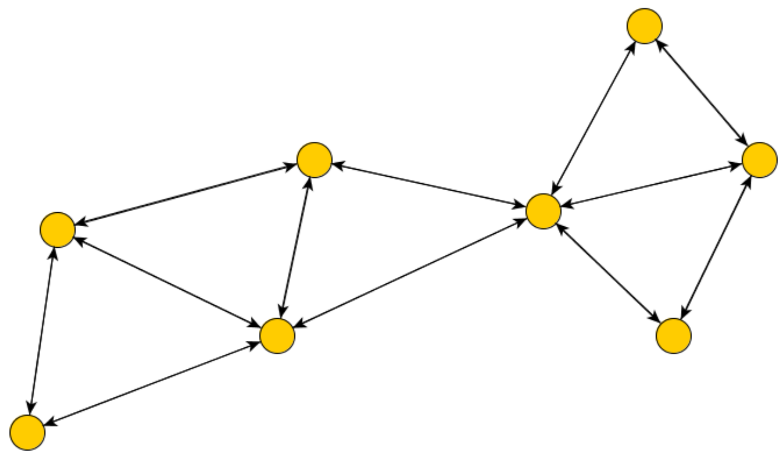
- Where is an agent likely to go?
  - Which path is best?

Mental Maps

- Represent the environment as a graph
  - Nodes represent locations
  - Edges represent paths
- Agents interpret the environment differently



Regular Grid
(Raster Image)



Vector Data

Fuzzy Weighted Graphs

- Each path segment has several measurable features (distance, slope, path quality, …)
  - Values are fuzzy numbers to capture uncertainty

- Each agent has an individual interpretation of the cost of travel associated with each feature
  - Example: An more athletic agent assigns a low cost to a steep slope, whereas a less athletic agent assigns a higher cost.

Fuzzy Weighted Graphs

- Some notation…

$\widetilde{G} = (\mathcal{V}, \mathcal{E}, \mathcal{X})$	Fuzzy weighted graph
$\mathcal{V} = (v_1, \cdots, v_N)$	Set of vertices (Locations)
$\mathcal{E} = (e_1, \cdots, e_M)$	Set of edges (Paths)
$e_k = (v_i, v_j) \in \mathcal{V} \times \mathcal{V}$	Individual edge
$\widetilde{\boldsymbol{X}}(e_k) = (\widetilde{X}_1(e_k), \cdots, \widetilde{X}_r(e_k))$	Vector of features (Fuzzy numbers)
$\boldsymbol{p} = (e_1, \cdots, e_n) \in \mathcal{E}^n$	Continuous path
$\widetilde{\boldsymbol{F}}(\boldsymbol{p}) = \left(\widetilde{F}_1(\boldsymbol{p}), \cdots, \widetilde{F}_r(\boldsymbol{p})\right)$	Aggregated weight vector
$\widetilde{A}_i(\boldsymbol{p}) = \tilde{g}_i\left(\widetilde{F}_i(\boldsymbol{p})\right)$	Agent-specific interpretation
$\widetilde{\boldsymbol{A}}(\boldsymbol{p}) = \left(\widetilde{A}_1(\boldsymbol{p}) = \tilde{g}_1\left(\widetilde{F}_1(\boldsymbol{p})\right), \cdots, \widetilde{A}_r(\boldsymbol{p}) = \tilde{g}_r\left(\widetilde{F}_r(\boldsymbol{p})\right)\right)$	Agent interpretation vector

Picking a Route

- An agent uses the agent interpretation vector from each path to plan a route to the goal

- An optimal, rational agent would always pick the path with the lowest perceived cost

BUT!

- Agents do not always make optimal decisions
  – Limited knowledge
  – Unable to think about many things at once

Bounded Rationality

- Agents have limited resources with which to make decisions
  - Time, memory, intelligence
- They consider only a subset of all available factors,
  - Known as Bounded Rationality (H. Simon)
  - Factor costs are fused in a nonlinear way

OWA for Bounded Rationality

- Ordered Weighted Average (OWA) operators
  - Aggregates the interpreted costs of each factor
  - Parameterized class of mean operator
- Operation:
  - Define a vector of weights
  - Sort the cost values
  - Assign weights in the new order
- Can model decisions like “at least two are big”
  - Example: (0.5, 0.5, 0, 0, 0) = average of top two

Extension to Fuzzy Sets

$$\Phi_{\widetilde{\boldsymbol{W}}} : (\widetilde{A}_1(\boldsymbol{p}), \cdots, \widetilde{A}_r(\boldsymbol{p})) \mapsto \widetilde{C}(\boldsymbol{p})$$
$$\widetilde{\boldsymbol{W}} = (\widetilde{W}_1, \cdots, \widetilde{W}_r)$$
$$\widetilde{W}_i : \mathbb{R} \to [0, 1]$$

For each $\alpha \in [0, 1]$,

$${}^{\alpha}\Phi_{\widetilde{\boldsymbol{W}}}({}^{\alpha}\widetilde{A}_1(\boldsymbol{p}), \cdots, {}^{\alpha}\widetilde{A}_r(\boldsymbol{p})) = \left(\left. \frac{\sum_{i=1}^{r} w_i a_{\sigma(i)}}{\sum_{i=1}^{r} w_i} \right|_{\substack{w_i \in {}^{\alpha}\widetilde{W}_i \\ a_i \in {}^{\alpha}\widetilde{A}_i(\boldsymbol{p}) \\ i=1,\cdots,r}} \right),$$

$$\begin{aligned} \text{where} \quad & \sigma : (1, \cdots, r) \to (1, \cdots, r) \\ \text{such that} \quad & a_{\sigma(i)} \geq a_{\sigma(i+1)} \ \forall\, i = 1, \cdots, r-1 \end{aligned}$$

From the set of ${}^{\alpha}\Phi_{\widetilde{\boldsymbol{W}}}$, the final cost value can be obtained as

$$\widetilde{C}(\boldsymbol{p}) = \bigcup_{0 \leq \alpha \leq 1} \alpha \cdot {}^{\alpha}\Phi_{\widetilde{\boldsymbol{W}}}({}^{\alpha}\widetilde{A}_1(\boldsymbol{p}), \cdots, {}^{\alpha}\widetilde{A}_r(\boldsymbol{p})).$$

Fast Implementation

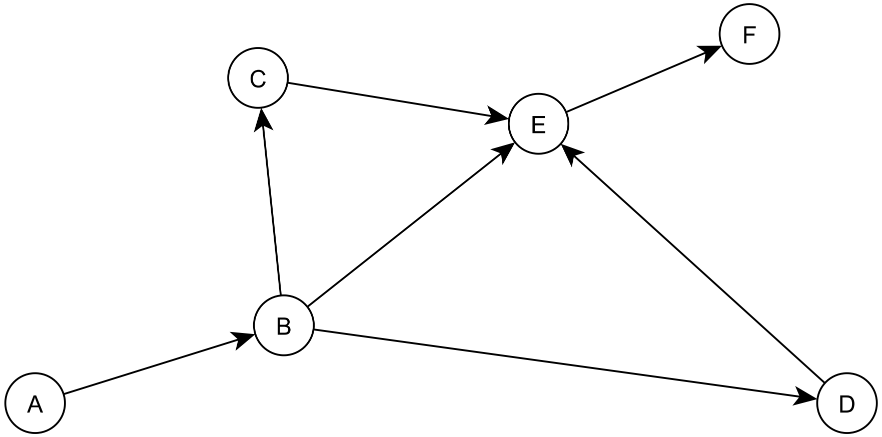
- For normal, convex fuzzy numbers, the α-level OWA operator can be quickly computed using the interval endpoints

Zhou S., Chiclana F., John R.I., Garibaldi J.M.: Alpha-Level Aggregation: A Practical Approach to Type-1 OWA Operation for Aggregating Uncertain Information with Applications to Breast Cancer Treatments. IEEE Transactions on Knowledge and Data Engineering, vol. 23, no. 10, pp. 1455–1468 (2011).

Example Scenario

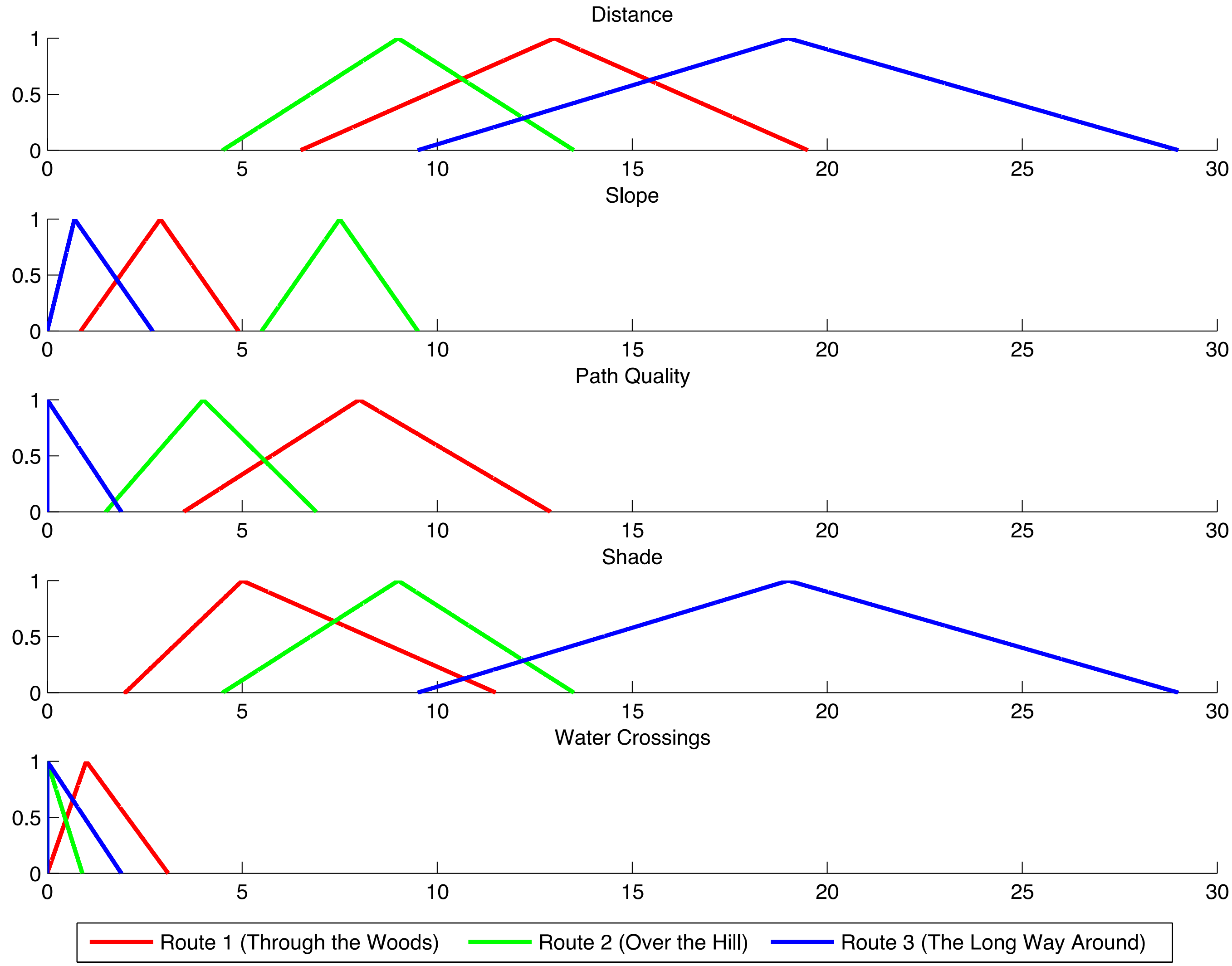
Evaluate Edge Features

Environment is represented as a directed graph with triangular fuzzy number feature values



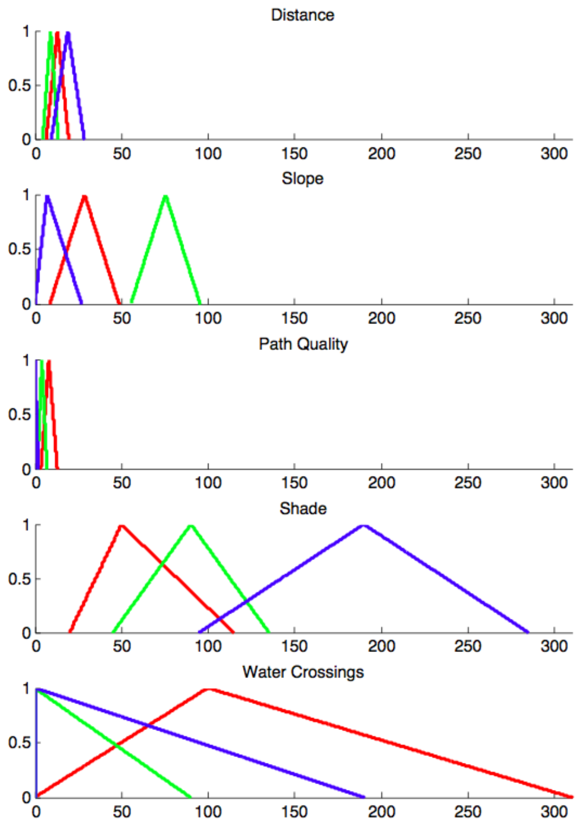
Edge	Distance	Slope	Path	Shade	Water
(A, B)	$(1, 2, 3)$	$(0, 0.64, 2.6)$	$(0, 0, 0.2)$	$(1, 2, 3)$	$(0, 0, 0.2)$
(B, C)	$(2, 4, 6)$	$(0.8, 2.8, 4.8)$	$(1.5, 3.5, 5.5)$	$(0, 0.5, 2.5)$	$(0, 0, 0.4)$
(B, D)	$(3.5, 7, 11)$	$(0, 0.57, 2.6)$	$(0, 0, 0.7)$	$(3.5, 7, 11)$	$(0, 0, 0.7)$
(B, E)	$(2.5, 5, 7.5)$	$(5.5, 7.5, 9.5)$	$(1.5, 4, 6.5)$	$(2.5, 5, 7.5)$	$(0, 0, 0.5)$
(C, E)	$(2.5, 5, 7.5)$	$(0.86, 2.9, 4.9)$	$(2, 4.5, 7)$	$(0, 0.5, 3)$	$(0, 1, 2.3)$
(D, E)	$(4, 8, 12)$	$(0, 0.7, 2.7)$	$(0, 0, 0.8)$	$(4, 8, 12)$	$(0, 0, 0.8)$
(E, F)	$(1, 2, 3)$	$(0, 0.25, 2.3)$	$(0, 0, 0.2)$	$(1, 2, 3)$	$(0, 0, 0.2)$

Aggregate Edge Features

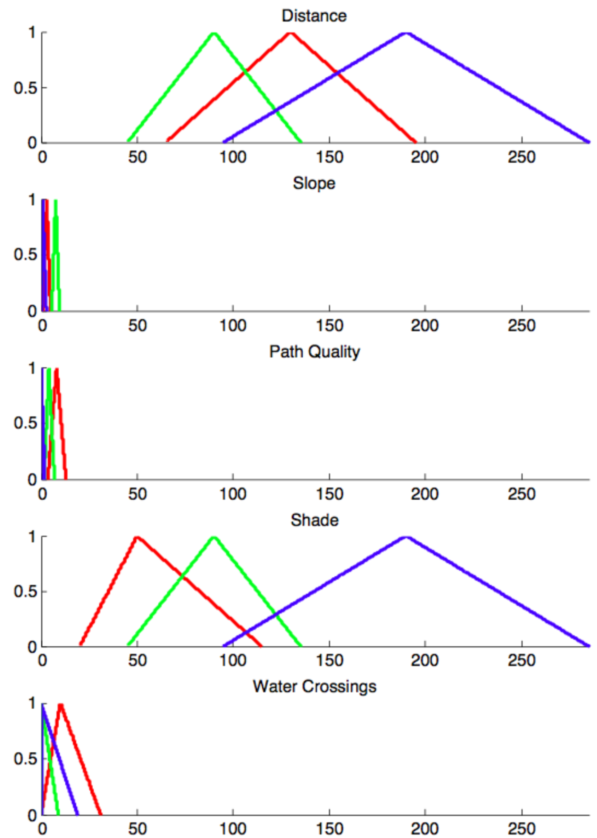


Agent Interpretations

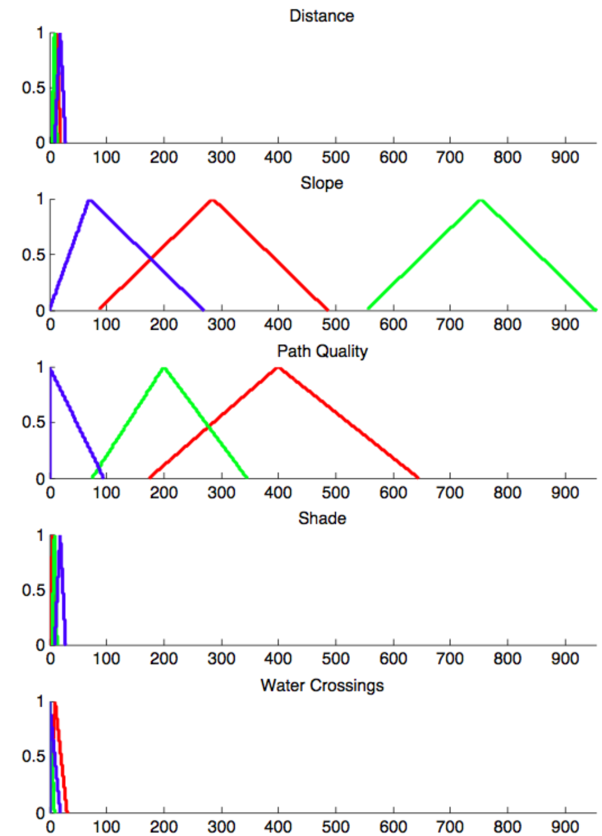
Agent 1



Agent 2



Agent 3

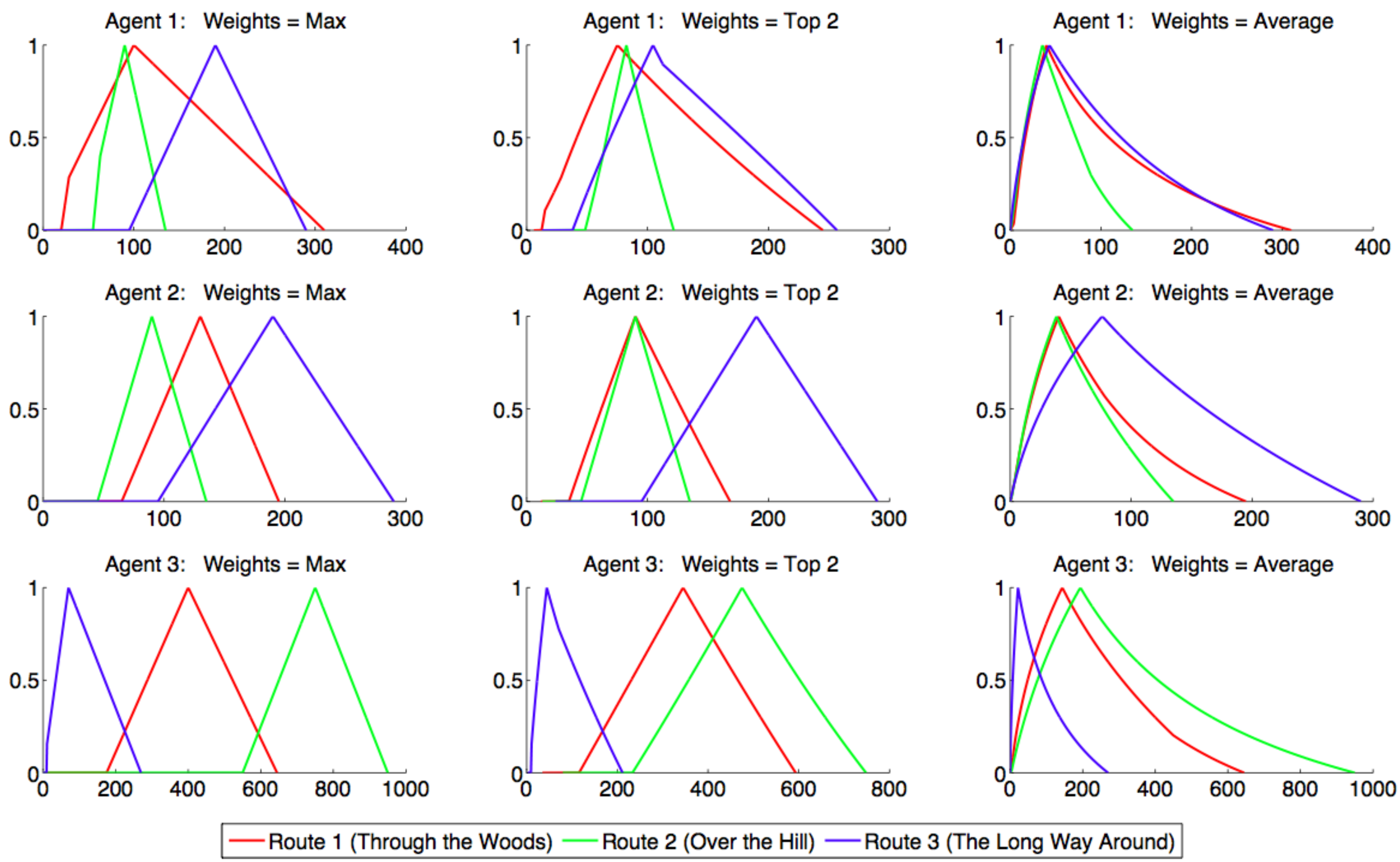


Define OWA Weights

- Compare three different OWA operators:
  - Maximum
  - Average of top 2 features
  - Average over all features

	$\widetilde{W}_1$	$\widetilde{W}_2$	$\widetilde{W}_3$	$\widetilde{W}_4$	$\widetilde{W}_5$
$\widetilde{\boldsymbol{W}}_{(\text{Max})}$	$(0, 0.5, 1)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
$\widetilde{\boldsymbol{W}}_{(\text{Top 2})}$	$(0.5, 1, 1)$	$(0.5, 1, 1)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
$\widetilde{\boldsymbol{W}}_{(\text{Average})}$	$(0, 0.2, 0.4)$	$(0, 0.2, 0.4)$	$(0, 0.2, 0.4)$	$(0, 0.2, 0.4)$	$(0, 0.2, 0.4)$

Path Cost Evaluation



Ranking

- Rank the paths based on the evaluated costs
  - Can use any appropriate ranking method

- Useful to use an index that allows for an optimism/pessimism parameter
  - Liou and Wang
  - Optimistic agents look at the best possible outcome
  - Pessimistic agents look at the worst possible outcome

Liou, T., Wang, M.J.: Ranking fuzzy numbers with integral value. Fuzzy Sets and Systems, vol. 50, no. 3, pp. 247–255 (1992).

Future Work

- Extend the path evaluation into a path-planning algorithm
  - Look at fuzzy shortest-path algorithms that return multiple results
- Develop a general agent movement model
  - Use the path planning algorithm and follow a least-cost route

Path Planning Model

Elevation

157	145	66		57	116	61	19	4	1
123	98	40	11	22	54	34	11	3	1
117	95	54	9	13	24	23	13	3	0
139	136	126	121	114	100	104	47	7	2
156	170	193	240	219	196	167	95	15	1
144	161	194	211	209	184	164	79	11	1
109	117	126	115	78	60	76	31	4	0
86	74	52	12	3	8	14	9	9	20
105	66	23	2	4	31	48	24	33	87
151	96	21		17	86	111	56	51	102

Shaded

Shaded	Shaded	Shaded		No Shade	No Shade	No Shade	No Shade	No Shade	No Shade
Shaded	Shaded	Shaded	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade
Shaded	Shaded	Shaded	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade
Shaded	Shaded	Shaded	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade
Shaded	Shaded	Shaded	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade
Shaded	Shaded	Shaded	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade
Shaded	Shaded	Shaded	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade
Shaded	Shaded	Shaded	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade
Shaded	Shaded	Shaded	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade	No Shade
Shaded	Shaded	Shaded		No Shade	No Shade	No Shade	No Shade	No Shade	No Shade

Road Quality

Offroad	Offroad	Offroad		Offroad	Offroad	Offroad	Offroad	Offroad	Offroad
Offroad	Offroad	Offroad	Paved	Offroad	Offroad	Offroad	Offroad	Offroad	Offroad
Offroad	Dirt	Dirt	Paved	Paved	Paved	Paved	Paved	Paved	Offroad
Offroad	Dirt	Offroad	Dirt	Offroad	Offroad	Offroad	Offroad	Paved	Offroad
Offroad	Dirt	Offroad	Dirt	Offroad	Offroad	Offroad	Offroad	Paved	Offroad
Offroad	Dirt	Offroad	Dirt	Offroad	Offroad	Offroad	Offroad	Paved	Offroad
Offroad	Dirt	Offroad	Dirt	Offroad	Offroad	Offroad	Offroad	Paved	Offroad
Offroad	Dirt	Dirt	Paved	Paved	Paved	Paved	Paved	Paved	Offroad
Offroad	Offroad	Offroad	Paved	Offroad	Offroad	Offroad	Offroad	Offroad	Offroad
Offroad	Offroad	Offroad		Offroad	Offroad	Offroad	Offroad	Offroad	Offroad

Water Crossing

No Water	No Water	No Water		No Water	No Water	No Water	No Water	No Water	No Water
No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water
No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water
No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water
Water	Water	Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water
No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water
No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water
No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water	No Water
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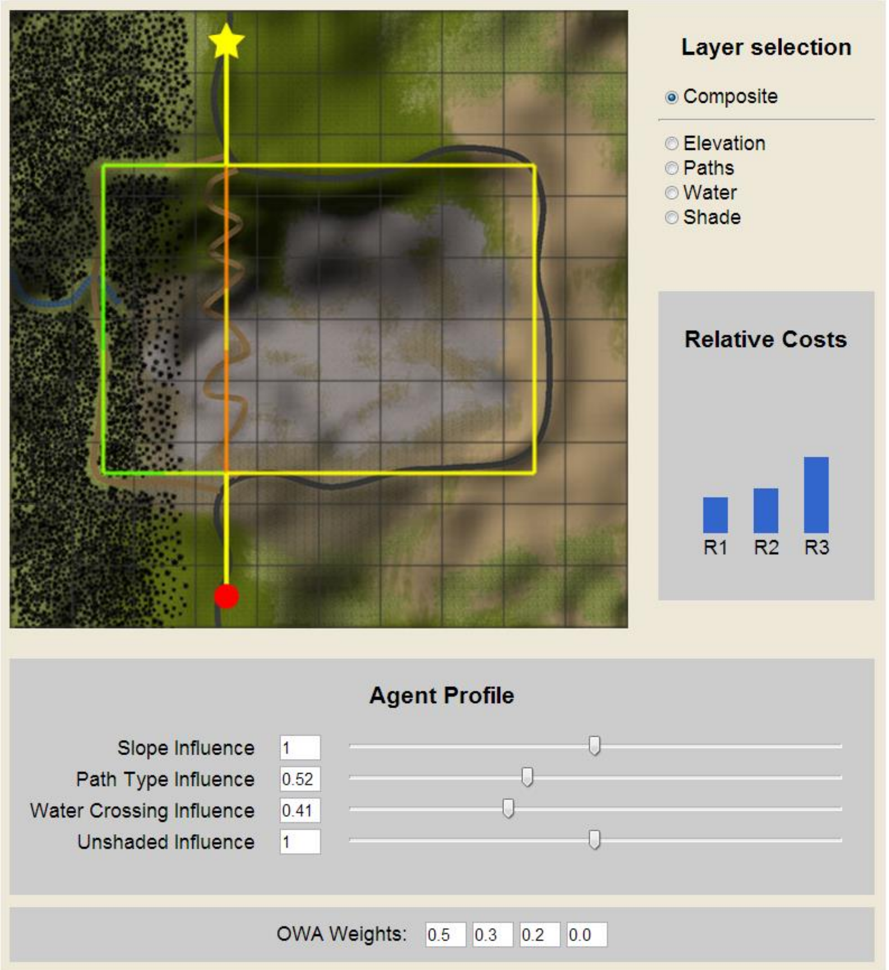
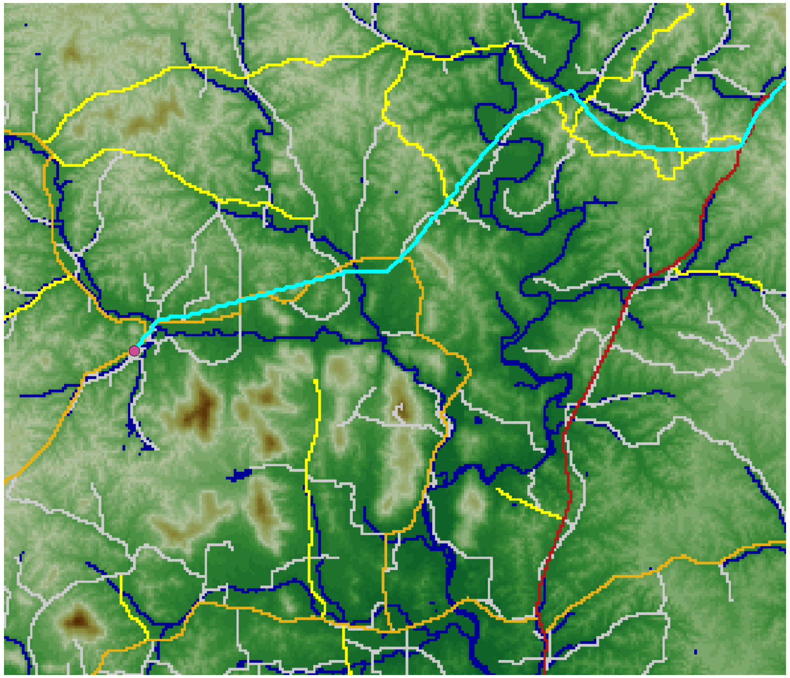
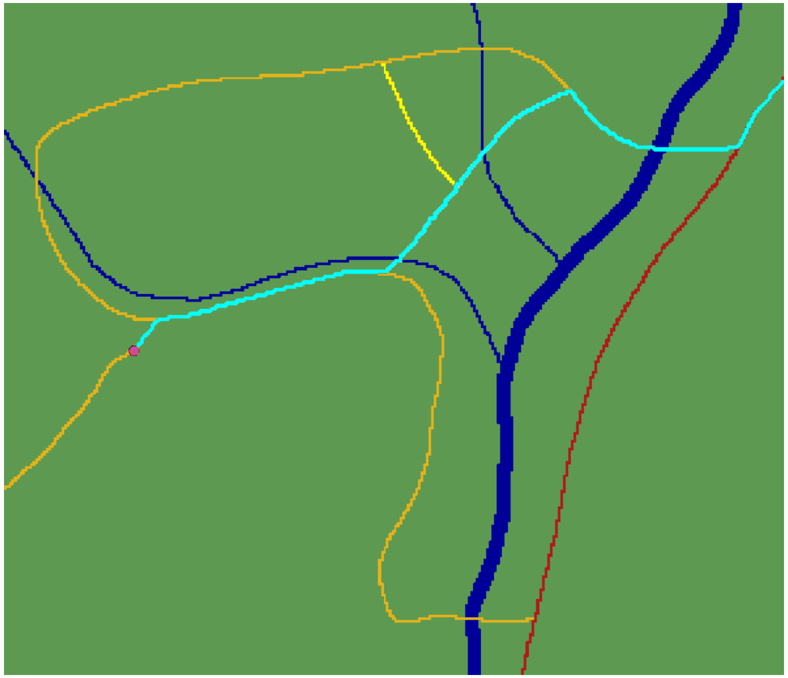


Mental Map Model

- The agent’s cost surface can be viewed as a mental map of the environment
  - Represents the agent’s knowledge
  - Can be updated and shared





Conclusions

- Fuzzy numbers are a natural way to represent the uncertainty in an environment
- The α-level OWA operator can be used to aggregate multiple features using bounded rationality
- Agent interpretation functions and weight vector can be adapted to various domains

Thank You