Object Set Matching With an Evolutionary Algorithm

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IEEE CISDA 2011 April 15, 2011 Paris, France

Outline

- Spatial Reasoning
  - Histograms of Forces
  - Comparing Histograms and Sets of Histograms
  - Elastic Angles
- Search Strategy
  - An Evolutionary Algorithm
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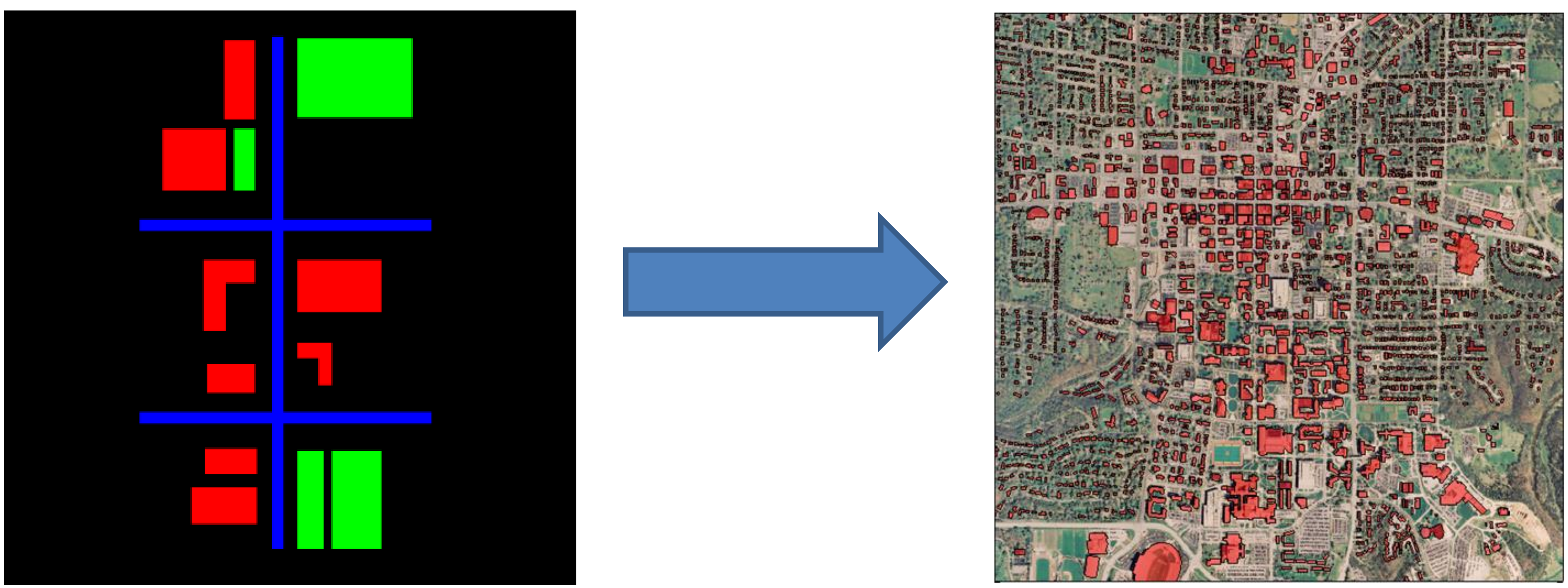
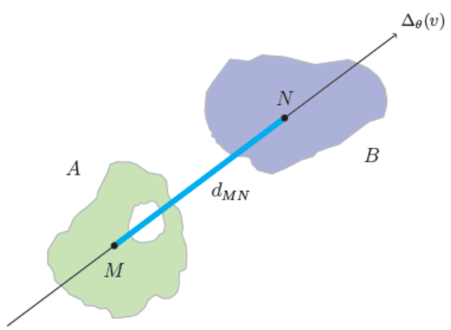
Spatial Reasoning

– Where am I?
  - "I see a long rectangular building on my left and a small L-shaped building on my right."
– Can I draw a map?

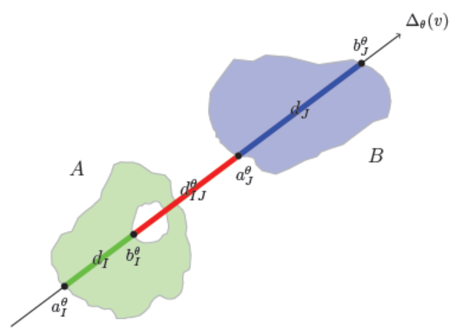


Histograms of Forces

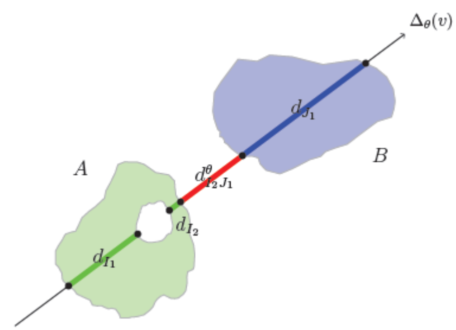
– A way of representing the degree of truth of the statement, “A is in direction θ from B.”



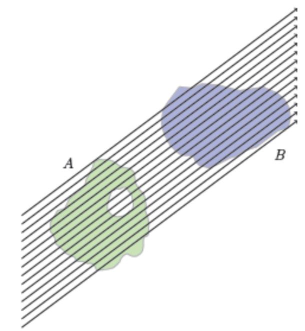
Points



Line Segments



Longitudinal Sections



Directions

$$\phi_r(M-N) = \frac{1}{d_{MN}^{r}}$$

$$(\theta, A_\theta(v), B_\theta(v)) = \sum_{i,j} f_r\left(d_{I_i}, d_{I_iJ_j}^{\theta}, d_{J_j}\right)$$

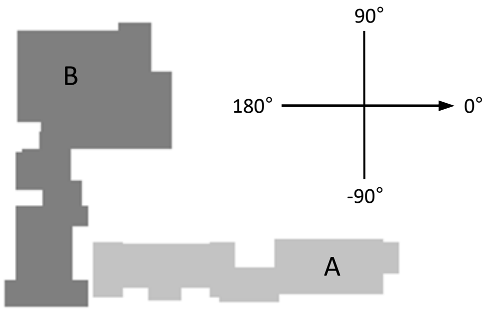
$$f_r\left(d_I, d_{IJ}^{\theta}, d_J\right) = \int_{a_I^\theta}^{b_I^\theta} \int_{a_J^\theta}^{b_J^\theta} \phi_r(u-v)\,dv\,du$$

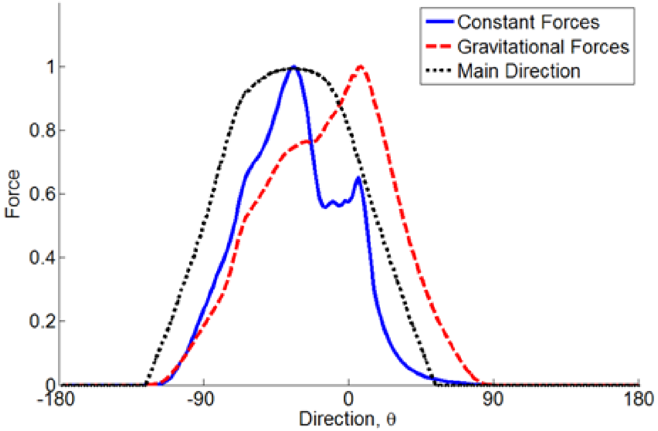
$$F_r^{AB}(\theta) = \int_{-\infty}^{\infty} \mathcal{F}_r(\theta, A_\theta(v), B_\theta(v))\,dv$$

Histograms of Forces

– Evaluate with different values of *r*
  - *r* = 0 gives constant forces invariant to distance
  - *r* = 2 gives gravitational forces invariant to scale
– Combine to get the main direction histogram

$$a^{AB}(\theta) = \max\{a_0^{AB}(\theta), min\{a_2^{AB}(\theta), b_0^{AB}(\theta)\}\}$$





Comparing Histograms

- Normalize histograms

$$\overline{F_r^{AB}}(\theta) = m^{-1} F_r^{AB}(\theta + c)$$

  - *m* is the mean value
  - *c* is the centroid

- Compute the cross-correlation

$$\mu_C(F_1, F_2) = \frac{\sum_\theta F_1(\theta) F_2(\theta)}{\sqrt{\sum_\theta F_1^2(\theta)}\sqrt{\sum_\theta F_2^2(\theta)}}$$

Comparing Sets of Histograms

- Given two sets of objects
  - $\mathcal{S} = \{o_1, o_2, \ldots, o_N\}$
  - $\mathcal{C} = \{c_1, c_2, \ldots, c_N\}$

- Build the set of force histogram relationships

$$\mathcal{F}^{AB} = \{F_0^{AB},\ F_2^{AB}\}$$

$$\mathcal{H}_S = \left\{\mathcal{F}^{o_i o_j} \,\middle|\, (o_i, o_j) \in \mathcal{S} \times \mathcal{S} \,\middle|\, i < j\right\} = \left\{h_1^S, h_2^S, \ldots, h_M^S\right\}$$

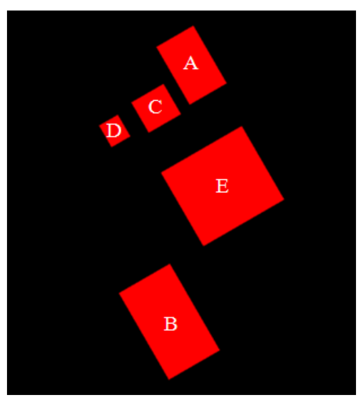
$$\mathcal{H}_C = \left\{\mathcal{F}^{c_i c_j} \,\middle|\, (c_i, c_j) \in \mathcal{C} \times \mathcal{C} \,\middle|\, i < j\right\} = \left\{h_1^C, h_2^C, \ldots, h_M^C\right\}$$

Comparing Sets of Histograms

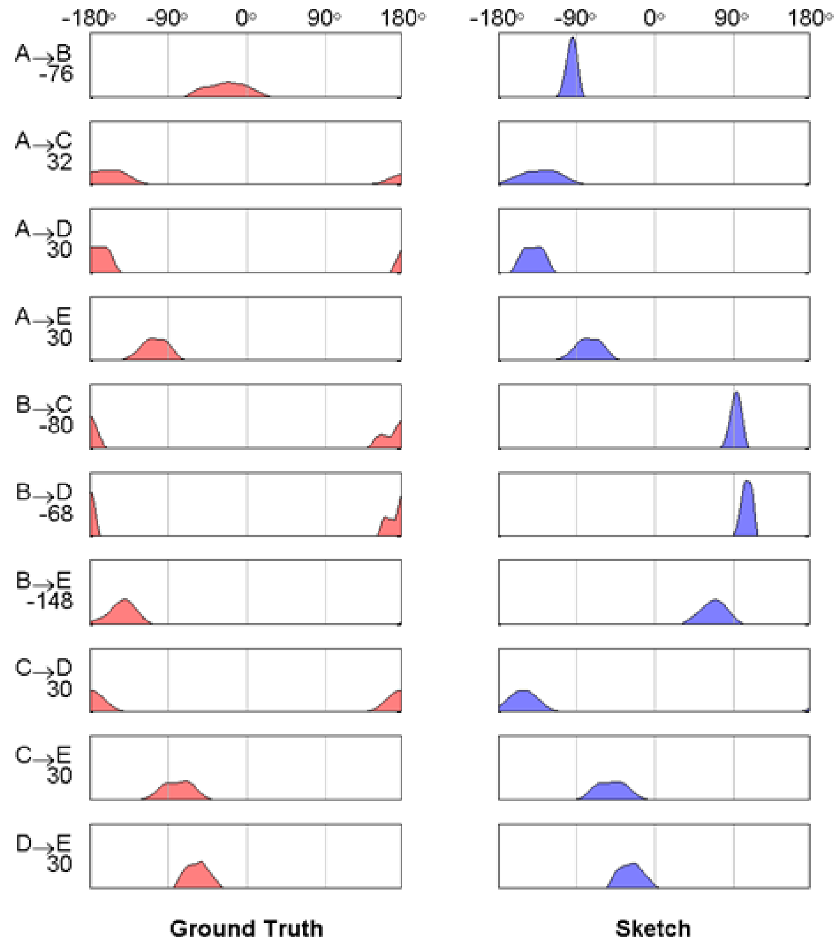
– Calculate the set of angle differences

$$D = \{centroid(h_i^S) - centroid(h_i^C) \mid 1 \le i \le M\}$$
$$= \{d_1, d_2, \dots, d_M\}$$

Ground Truth



Sketch



Comparing Sets of Histograms

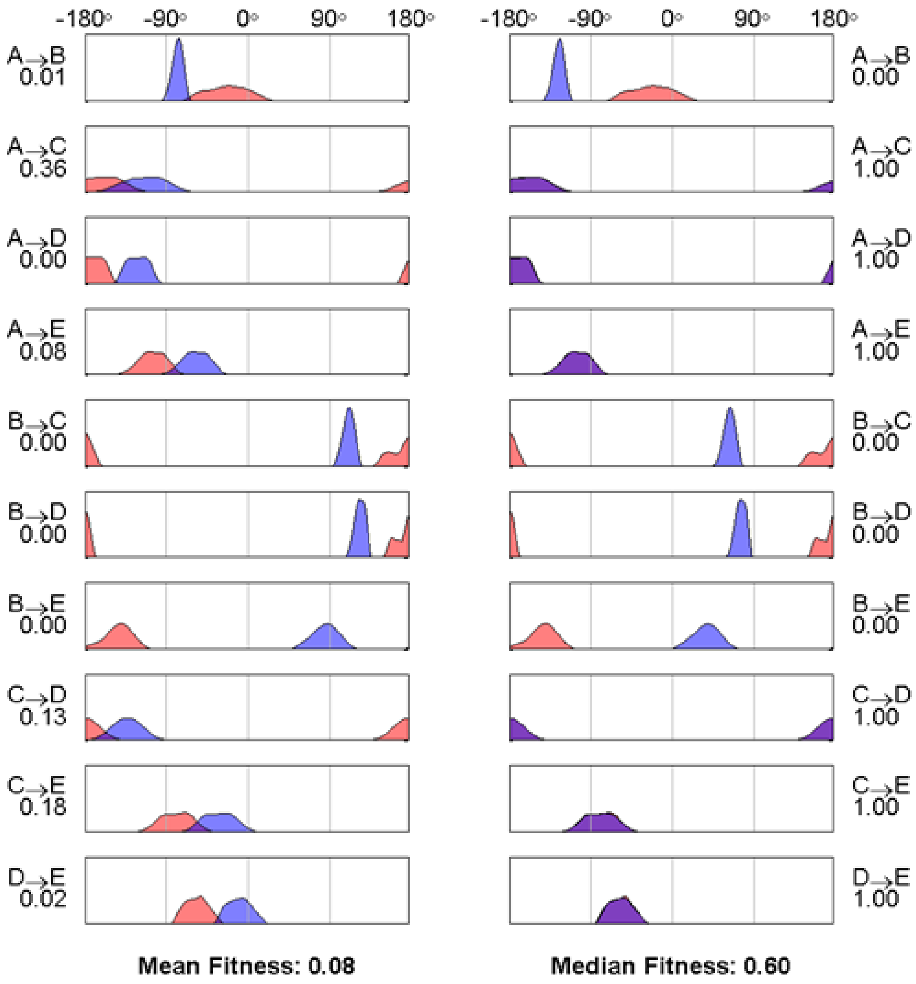
- Determine the optimal rotation angle of the sketch
  - Mean Angle = -4°
  - Median Angle = 30°

- Compute fitness as the average cross-correlation between histograms

$$\mu(\mathcal{H}_S, \mathcal{H}_C) = \frac{1}{M}\sum_{i=1}^{M} f\left[h_i^S(\theta - \theta_{Best}), h_i^C(\theta), \beta\right]$$

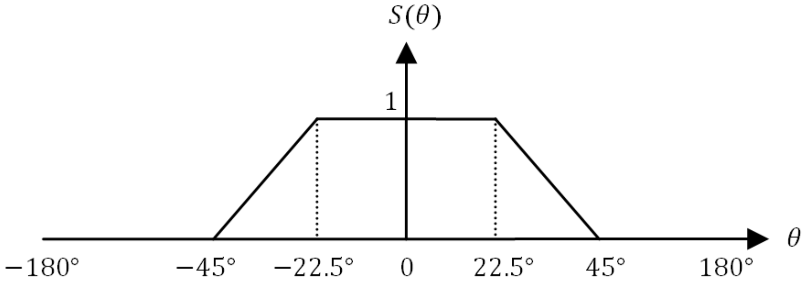
- Use a weighted average between the constant and gravitational forces

$$f\left(\mathcal{F}^{AB}, \mathcal{F}^{A'B'}, \beta\right) = \beta\mu_C\left(F_0^{AB}, F_0^{A'B'}\right) + (1-\beta)\mu_C\left(F_2^{AB}, F_2^{A'B'}\right)$$



Elastic Angles

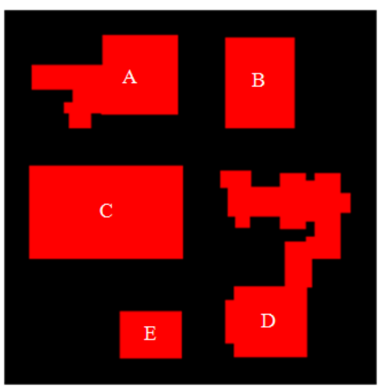
- Allows for small variations in the main directions
- Assume histograms are normalized within the database , along with their main directions
- Compute the cross-correlation of the normalized histograms with a weighting term



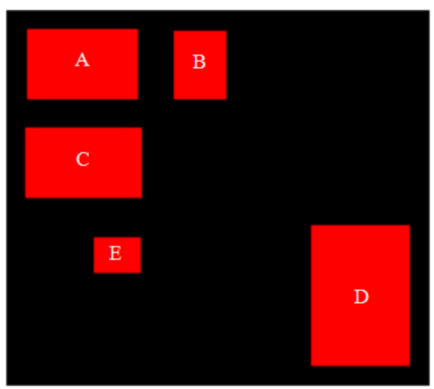
$$\mu_{Elastic}(\mathcal{H}_S, \mathcal{H}_C) = \frac{1}{M}\sum_{i=1}^{M} f\left(\overline{h_i^S}, \overline{h_i^C}, \beta\right) S(\theta_{Best} - d_i)$$

Elastic Angles

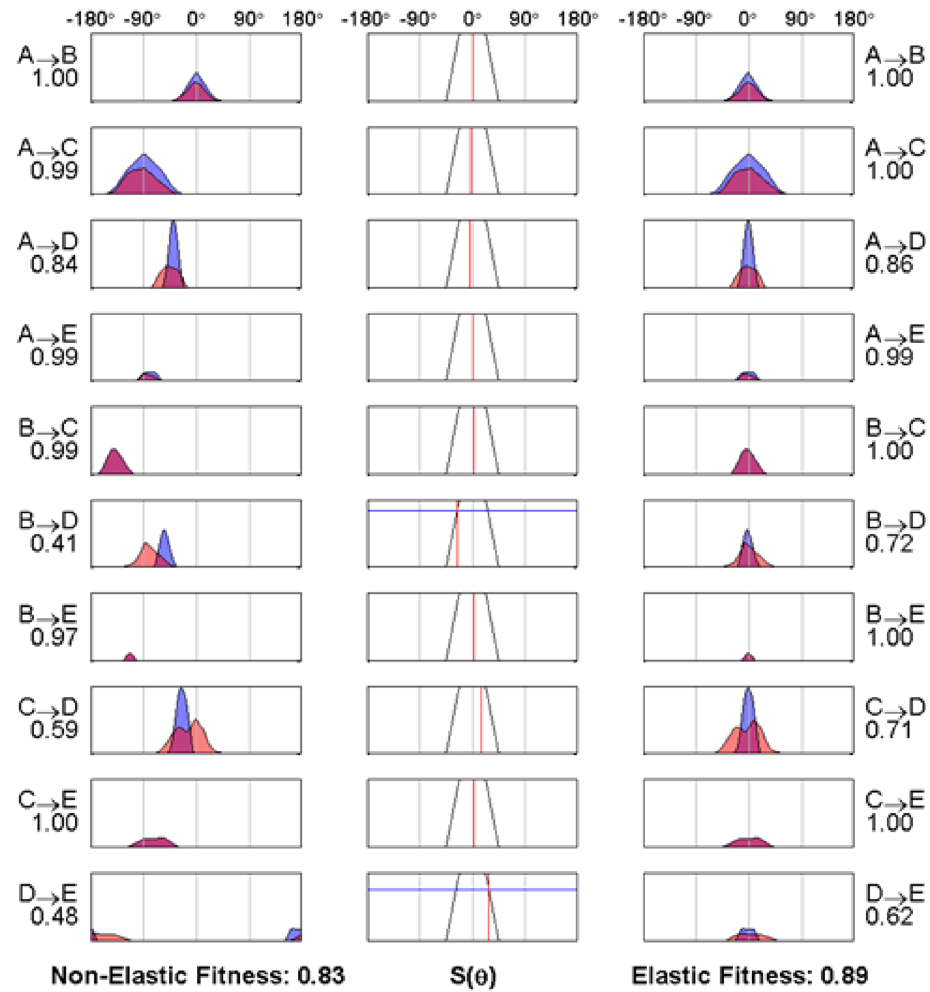
- Example where a building has been shifted down and to the right



Ground Truth

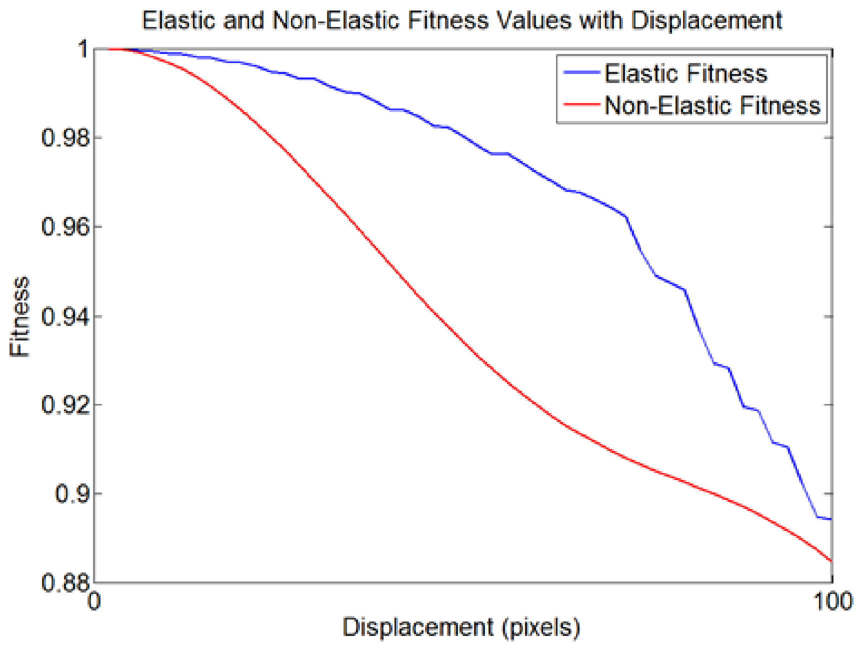


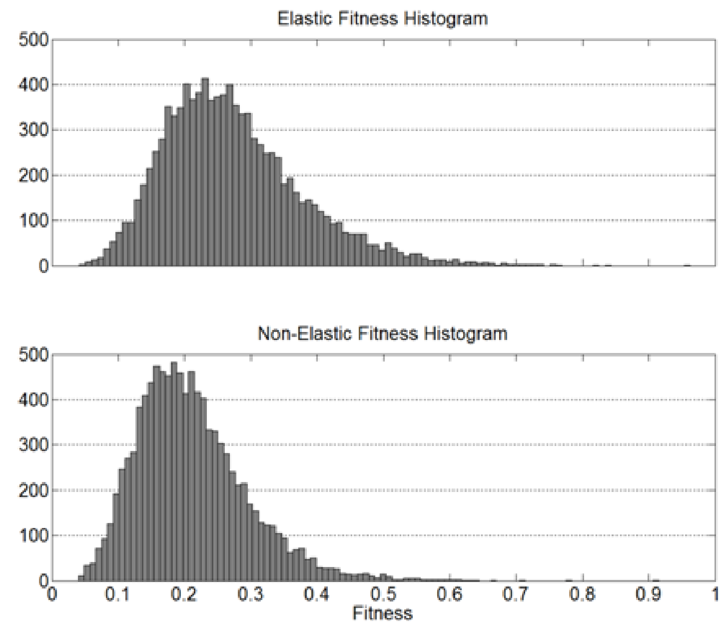
Sketch



Elastic Angles

- Typically increases individual fitness values
- Does this for all sets of objects, raising the overall average fitness value





An Evolutionary Algorithm

- Given the reference set and the input sketch
- Create random chromosomes for the population
- Define a mutation operator
  - Replace one building with the best fit

Algorithm I: Evolutionary Matching Algorithm

Input: $\mathcal{H}_R$ and $\mathcal{H}_S$

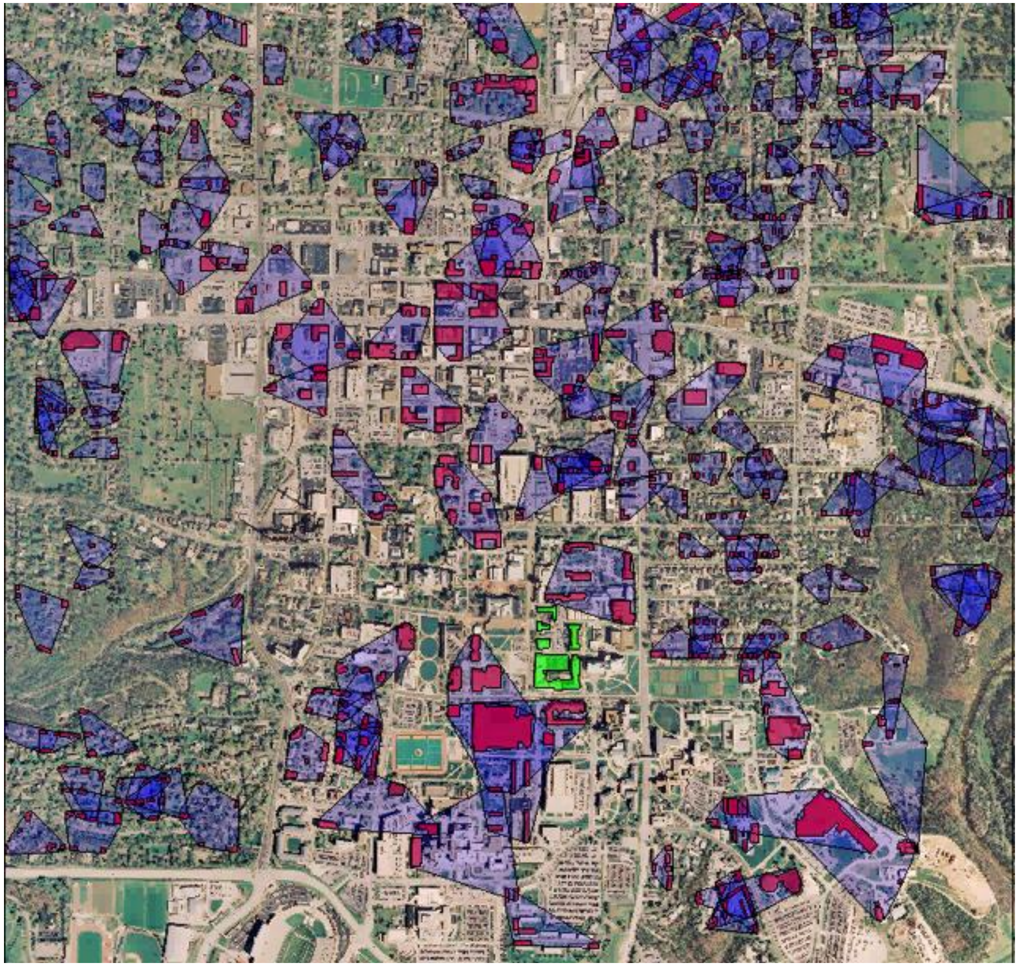
Create an initial population of chromosomes, P
Set generation counter $t = 0$
while stopping criteria is not met
    for each chromosome, $\mathcal{C}_{P}$, in P
        Mutate $\mathcal{C}_P$ and create $\mathcal{C}_C$
        Replace $\mathcal{C}_P$ with $\mathcal{C}_C$ probabilistically
    end

    if t is a multiple of $t_{Repopulate}$
        Replace the least fit members of P with new random chromosomes
    end

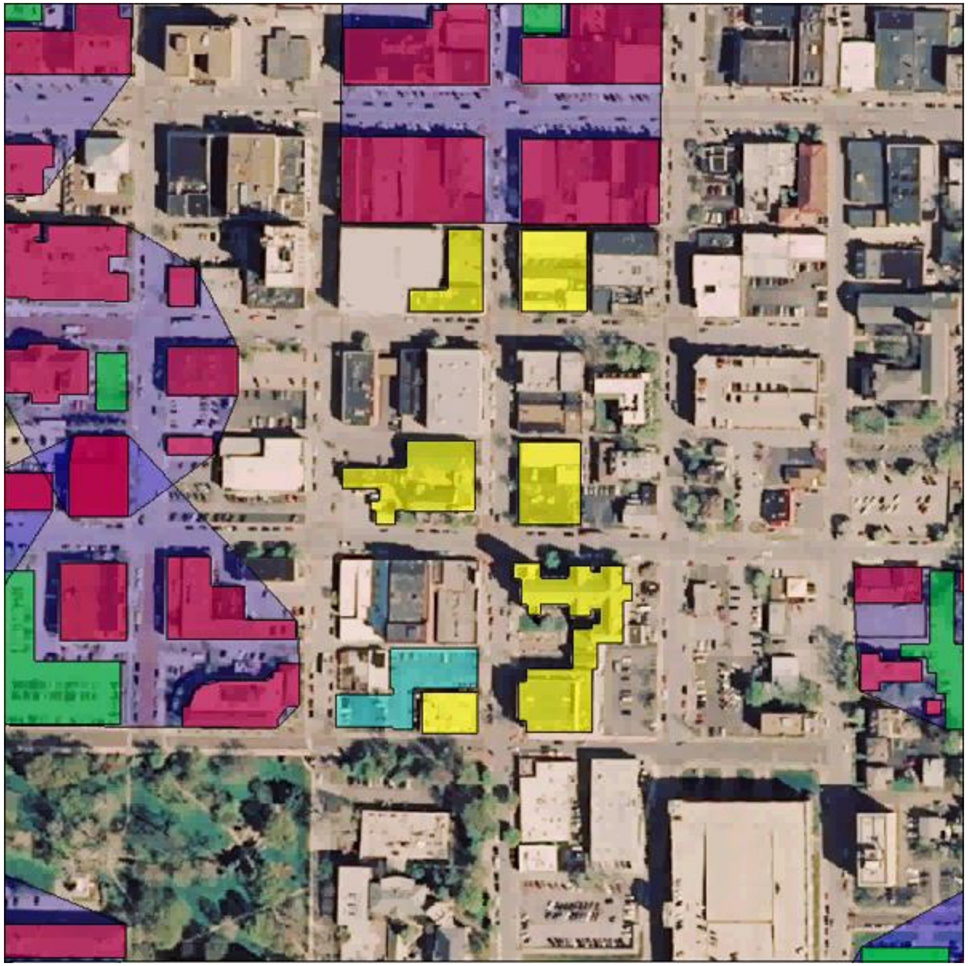
    $t = t + 1$
end

Output: Top chromosomes in P

An Evolutionary Algorithm



An Evolutionary Algorithm



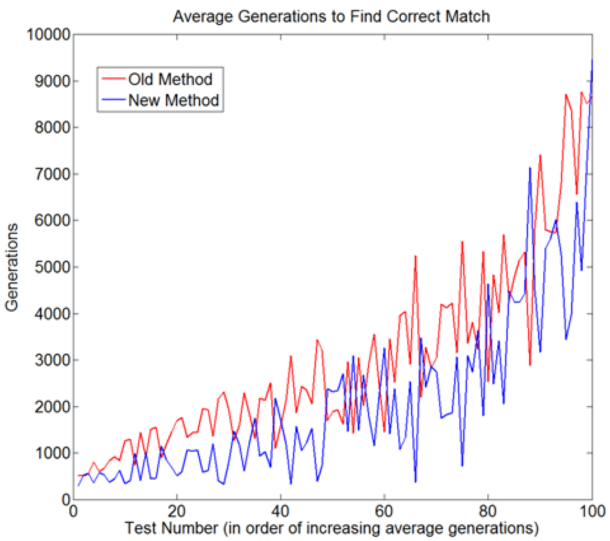
Experiments

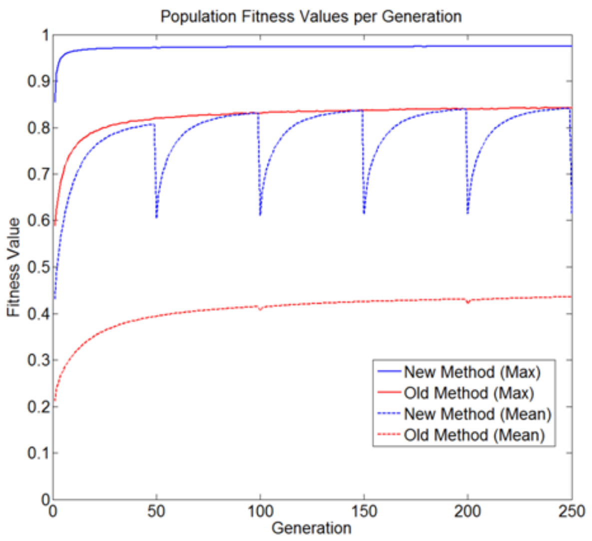
- Created 100 random test sets of 5 nearby buildings, simplified and rotated to a random angle
- Ran each method 30 times and recorded the average number of generations to recover the original buildings

	Evolutionary Algorithm Parameters	
	Old Method	***New Method***
Max Generations	10,000	10,000
Population Size	493	493
Replacement Frequency	100 Generations	50 Generations
Percent of Replacement	10%	50%
Histogram Bias, β	1	0.5
Rotation Method	Mean Angle	Median Angle
Elastic Angles	Non-Elastic	Elastic

Results

	Matching Results	
	Old Method	***New Method***
Percent of tests which found correct match	89.4%	95.1%
Average Generations	3001	2104





Recent Developments

- New algorithms give faster convergence
  - Replace multiple buildings each mutation
    - Keep only one or two buildings from the chromosome and pick the best possible buildings for the rest
  - Sub-Graph Isomorphism
    - Allows entire search space to be examined
    - Difficult to scale
  - Hybrid approach
    - Use sub-graph isomorphism as a local mutation operator within larger EA framework

Future Work

- Adding labels to objects such as *parking lot*, *church*, *office building*, *restaurant*
- Integrate road networks and other contextual information
- Make optimal assignment between buildings of the sketch and chromosome

Conclusion

- Matching sets of objects using spatial reasoning techniques has intriguing intelligence applications
  - Helps answer “Where am I?” or “Where are they?”
- Very large search space can be pruned by looking for sets with unique features
- Force histograms are a robust spatial descriptor which can accommodate imprecise sketches

Questions?